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# Analysis of the magneto-optical properties of inhomogeneously magnetized systems

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**Abstract.** A theory is developed for calculating the normal incidence polar Kerr effect in multilayered systems that are optically isotropic but magnetically inhomogeneous in a direction parallel to the direction of propagation of the probing radiation. Specific examples are presented for well known limiting cases and situations that may be commonly found in multilayered systems that are associated with magneto-optic recording media. A particular practical example is given for the Co–Pd multilayer system where the consequences of inhomogeneously polarized Pd layers can be seen directly from *in situ* experimental data and interpreted using the theoretical analysis. Finally, an analysis is given for the optimization of layer thicknesses for the Co–Pd system that leads to structures that have the optimum intrinsic magneto-optic parameter that will ensure the best performance in magneto-optic readout systems.

## 1. Introduction

There is increasing interest in magnetic multilayers for magneto-optic recording that utilize the advantages of materials that possess increased magneto-optic performance by virtue of the combination and mutual proximity of materials that may or may not be intrinsically magnetic at normal temperatures. The increased or altered activity can be the result of new magneto-optic transitions that may be a consequence of quantum-well structures [1–5] or may be the result of non-magnetic media becoming polarized when in proximity to ferromagnetic media, as occurs at the interfaces of multilayered structures [6]. In the former case prediction of absolute values of fundamental material parameters and the subsequent behaviour of layered systems is not easy. In the latter case, the prediction of new magneto-optic effects is equally difficult and the calculation of the measurable Kerr or Faraday effects is complicated by the fact that the magnetization distribution may be non-uniform within a particular layer.

In this paper a theoretical analysis is given for dealing with the calculation of the polar magneto-optical Kerr effect [7] in inhomogeneously magnetized multilayers where the material parameters are known together with their variation in a direction perpendicular to the planes containing the film interfaces. A number of specific cases are treated as examples and the results of a recent experiment on Co–Pd multilayers [8] are summarized that illustrate the existence of inhomogeneous magnetization in Pd and its effect on the evolving magneto-optical Kerr effect of this system. From these data the basic material parameters have been determined and used to calculate the optimum structures that will lead to maximized magneto-optic performance for the purposes of information readout in magneto-optic recording.

## 2. Theoretical analysis

The calculation of magneto-optical properties of multilayered systems can, in principle, always be performed using the well known $4 \times 4$ characteristic matrix method for dealing with multilayered systems [9]. Where each layer is homogeneous, this is a preferred method of dealing with linear magneto-optical calculations and is quite general in that all the principal magneto-optic orientations (polar, longitudinal and transverse [7]) can be treated. In situations where the individual layers are themselves inhomogeneous the calculation may still be carried out though, in such cases, each layer may have to be subdivided into a large number of elemental layers, each being different for the rest but homogeneous. Such an approach makes the calculation rather tedious and time consuming. An alternative matrix approach was developed earlier [10] where the characteristic matrix for an inhomogeneously magnetized layer was derived and could be applied identically to the usual $4 \times 4$ matrix approach. However, the method is not simple and requires considerable computation in forming the specific matrix for a layer of particular magnetization distribution.

Apart from the complex and tedious nature of such calculations, physical insight into the magneto-optic behaviour of inhomogeneously magnetized systems is difficult to appreciate without simple analytical expressions that are able to throw light on how the magneto-optical effects are produced and vary in such systems.

In the following analysis a relatively simple formulation is derived for dealing with the normal incidence polar Kerr effect, specifically for any multilayered system provided that its optical properties are approximately uniform throughout. The later requirement may, at first, seem excessively restrictive but in practice it is a condition that is often satisfied. This is particularly the case in next generation magneto-optic recording media such as Co–Pd and Co–Pt [11–13].

The theoretical framework is based around two fundamental principles. The first is the principle of the superposition of linear magneto-optical effects [14]. This states that 'The magneto-optically induced electric field components resulting from an interaction of electromagnetic radiation with a magnetic multilayered system are linear superpositions of the complex magneto-optic fields due to individual magnetic layers of the system where the fields associated with each are calculated on the basis that the rest of the system is non-magnetic'. The second is the application of the concept of differential reflectance applied to the calculation of the magneto-optic Kerr coefficient $k$ [15]. It is well known that, at normal incidence, for the polar Kerr configuration where the magnetization is along the surface normal, radiation may propagate in the medium as two circularly polarized modes that have opposite senses (right handed and left handed) of rotation of the electric field vector. The effective refractive indices that the material presents for these two modes are designated $n_+$ and $n_-$ where $n_\pm = n(1 \pm Q/2)$ [15]. The parameters $n$ and $Q$ are the complex isotropic refractive index and magneto-optic Voigt $Q$-parameter, respectively, that appear in the skew-symmetric permittivity tensor for a gyroelectric medium [7]. It is well known and easy to show for any system that if the amplitude reflectances corresponding to these two modes are $r_+$ and $r_-$ the resulting Kerr coefficient $k$ is given by [15]

$$k = \mathrm{i}\frac{\delta r}{2} \tag{1}$$

where $\delta r = (r_+ - r_-)$ and is the differential system reflectance referred of course to the two refractive indices $n_\pm$, or rather the difference between them, $\delta n$ $(= n_+ - n_- = nQ)$. In cases that satisfy $k \ll r$, where $r$ $(= (r_+ + r_-)/2)$ is the isotropic amplitude reflectance, the complex Kerr rotation $\hat{\theta}$ is given by

$$\hat{\theta} = \theta + \mathrm{i}\varepsilon = \frac{k}{r}. \tag{2}$$

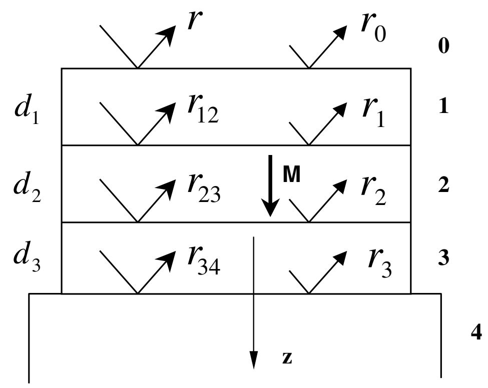


**Figure 1.** Reflectance amplitude coefficients associated with a three-layer system.

The problem that must be solved is the calculation of $k$ and $r$ for a situation where the $Q$-parameter is spatially dependent and where the isotropic refractive index is constant throughout the system.

Such a calculation is performed with respect to figure 1, showing a single system separated into three layers. The middle layer is regarded as uniformly magnetized in the positive $z$-direction according to the preferred sign convention for magneto-optic effects that has been defined previously [16]. With reference to figure 1 the Fresnel amplitude reflection coefficients at the four interfaces are given by the well known expression [17]

$$r_m = (n_m - n_{m+1})/(n_m + n_{m+1}) \qquad m = 0 \to 3 \tag{3}$$

where $n_0$ and $n_4$ are the indices of the incident and substrate media, respectively. In addition, $n_1 = n_3 = n$ and $n_2 = n_\pm$, the latter depending on the magneto-optical mode. The internal reflectances, taking into account the multiple reflections between layers, are obtained from the reiterative reflectance formula [18] given by

$$r_{m,m+1} = \frac{r_m + r_{m+1,m+2}\,\mathrm{e}^{\mathrm{i}2\rho_{m+1}}}{1 + r_m r_{m+1,m+2}\,\mathrm{e}^{\mathrm{i}\rho_{m+1}}} \tag{4}$$

where

$$\rho_m = \frac{2\pi n_m d_m}{\lambda} \tag{5}$$

is the complex phase thickness associated with the $m$th layer and, at the semi-infinite substrate interface, $r_{3,4} = r_3$. $\lambda$ is the free space wavelength. It is now necessary to determine the differential reflectances at each of these boundaries and ultimately therefore at the uppermost boundary of the system, that correspond to the differential index change $\delta n$ of the magnetic layer. Clearly the differential reflectance at the substrate interface $\delta r_3$ is zero since $r_3$ is independent of the modal indices. For the next interface the differential reflectance is obtained from

$$\delta r_{2,3} = \frac{\partial r_{2,3}}{\partial r_2}\delta r_2 + \frac{\partial r_{2,3}}{\partial \rho_3}\delta\rho_3 \tag{6}$$

where

$$\frac{\partial r_{2,3}}{\partial r_2} = 1 - r_3^2\,\mathrm{e}^{\mathrm{i}4\rho_3}. \tag{7}$$

Since layer three is independent of the modal indices, $\delta\rho_3$ is also zero. Moreover, it is easy to show from equation (3) and the definition of $n_\pm$, that $\delta r_2 = Q/2$. Hence,

$$\delta r_{2,3} = (1 - r_3^2\,\mathrm{e}^{\mathrm{i}4\rho_m})Q/2. \tag{8}$$

Similarly, for the next layer,

$$\delta r_{1,2} = \frac{\partial r_{1,2}}{\partial r_1}\delta r_1 + \frac{\partial r_{1,2}}{\partial r_{2,3}}\delta r_{2,3} + \frac{\partial r_{1,2}}{\partial \rho_2}\delta\rho_2 \tag{9}$$

where

$$\frac{\partial r_{1,2}}{\partial r_1} = 1 - r_{2,3}^2\,\mathrm{e}^{\mathrm{i}4\rho_2}$$

$$\frac{\partial r_{1,2}}{\partial r_{2,3}} = \mathrm{e}^{\mathrm{i}2\rho_2}$$

$$\frac{\partial r_{1,2}}{\partial \rho_2} = \mathrm{i}2r_3\,\mathrm{e}^{\mathrm{i}2(\rho_2+\rho_3)}.$$

Now, using equation (8), and since $\delta r_1 = -\delta r_2 = -Q/2$ and $\delta\rho_2 = Q\rho_m$,

$$\delta r_{1,2} = -(1 - r_3^2\,\mathrm{e}^{\mathrm{i}4(\rho_2+\rho_3)})Q/2 + (1 - r_3^2\,\mathrm{e}^{\mathrm{i}4\rho_3})\,\mathrm{e}^{\mathrm{i}2\rho_2}Q/2 + \mathrm{i}2r_3\rho_2 Q\,\mathrm{e}^{\mathrm{i}2(\rho_2+\rho_3)}. \tag{10}$$

Finally, for the incident interface the differential reflectance of the whole system is

$$\delta r = \frac{\partial r}{\partial r_{1,2}}\delta r_{1,2} \tag{11}$$

where

$$\frac{\partial r}{\partial r_{1,2}} = \mathrm{e}^{\mathrm{i}2\rho_1}(1 - r_0^2)/(1 + r_0 r_{1,2}\,\mathrm{e}^{\mathrm{i}2\rho_1})^2. \tag{12}$$

The subscripts on $\delta r$ have been omitted, since we are dealing with the system reflectance $r$. All other differential terms that might be attached to equation (11) are zero for reasons identical to those given above. Thus from equation (10)

$$\delta r = -\frac{Q}{2}\,\mathrm{e}^{\mathrm{i}2\rho_1}\frac{1 - r_0^2}{(1 + r_0 r_{1,2}\,\mathrm{e}^{\mathrm{i}2\rho_1})^2} \times[(1 - r_3^2\,\mathrm{e}^{\mathrm{i}4(\rho_2+\rho_3)}) - (1 - r_3^2\,\mathrm{e}^{\mathrm{i}4\rho_3})\,\mathrm{e}^{\mathrm{i}2\rho_2} - \mathrm{i}4r_3\rho_2\,\mathrm{e}^{\mathrm{i}2(\rho_2+\rho_3)}]. \tag{13}$$

From equations (1) and (13) and using the following identities, where it is assumed the incident medium is air,

$$\rho = \rho_1 + \rho_2 + \rho_3$$

$$1 - r_0^2 = 4n/(1 + n)^2$$

it follows that the Kerr coefficient due to the buried magnetic layer is given by

$$k = -\frac{\mathrm{i}nQ}{(1 + n)^2}\,\mathrm{e}^{\mathrm{i}2\rho_1}\frac{1}{(1 + r_0 r_3\,\mathrm{e}^{\mathrm{i}2\rho})^2} \times[(1 - r_3^2\,\mathrm{e}^{\mathrm{i}4(\rho_2+\rho_3)}) - (1 - r_3^2\,\mathrm{e}^{\mathrm{i}4\rho_3})\,\mathrm{e}^{\mathrm{i}2\rho_2} - \mathrm{i}4r_3\rho_2\,\mathrm{e}^{\mathrm{i}2(\rho_2+\rho_3)}]. \tag{14}$$

At this point it is worth reminding the reader that the $Q$-parameter may be spatially dependent and that, in order to apply the principal of superposition of magneto-optic effects, it is appropriate to consider the middle layer to be an elemental layer of thickness $\delta z$ buried at a depth $z$ in a film system of total thickness $d$. Consequently, allowing $d_2 \to \delta z$, $k \to \delta k$, $d_1 = z$, $d_3 = d - z$, and for convenience letting $\alpha = \mathrm{i}4\pi n/\lambda$, equation (14) may be written, to first order in $\delta z$,

$$\delta k = B_0\,\mathrm{e}^{\alpha z}Q[1 + B_1\,\mathrm{e}^{-2\alpha z} + B_2\,\mathrm{e}^{-\alpha z}]\alpha\delta z \tag{15}$$

where

$$B_0 = \frac{\mathrm{i}n}{(1+n)^2(1+r_0 r_3\,\mathrm{e}^{\mathrm{i}2\rho})^2} \tag{16}$$

$$B_1 = r_3^2\,\mathrm{e}^{2\alpha d} \tag{17}$$

$$B_2 = 2r_3\,\mathrm{e}^{\alpha d}. \tag{18}$$

Equation (15) is the important fundamental equation that allows one to deal with inhomogeneously magnetized systems and is used to perform integral calculations where the magneto-optic $Q$-parameter is any function $Q(z)$ of the $z$-co-ordinate. The general problem is therefore solved by integrating the expression

$$k = \int \mathrm{d}k = \int B_0\,\mathrm{e}^{\alpha z} Q(z)[1 + B_1\,\mathrm{e}^{-2\alpha z} + B_2\,\mathrm{e}^{-\alpha z}]\alpha\mathrm{d}z \tag{19}$$

over the thickness of the film structure. To illustrate the usefulness and validity of the expression a number of simple cases are considered below.

### *2.1. Case (I). Semi-infinite medium of homogeneous Q*

The case of a homogeneously magnetized, infinitely thick layer with air as the incident medium is well known and can be derived from the integration of equation (15). In this situation $d \to \infty$ and hence $B_1 = B_2 = 0$ and $B_0 = \mathrm{i}n/(1+n)^2$. Consequently, the Kerr coefficient $k$ is given by

$$k = \int_0^\infty \alpha Q B_0\,\mathrm{e}^{\alpha z}\,\mathrm{d}z = -QB_0 = -\mathrm{i}nQ/(1+n)^2. \tag{20}$$

In this simple case the Fresnel amplitude reflection coefficient is $r = (1-n)/(1+n)$ and results in the well known expression for complex Kerr rotation of

$$\hat{\theta} = k/r = -\mathrm{i}nQ/(1-n^2). \tag{21}$$

### *2.2. Case (II). Finite film having homogeneous Q*

For a film of finite thickness the $B$ coefficients, given by equations (16)–(18), are all finite, with $r_0$ and $r_3$ obtained from equation (3). It follows that

$$k = B_0\alpha Q\int_0^{d_0}[\mathrm{e}^{\alpha z} + B_1\,\mathrm{e}^{-\alpha z} + B_2]\mathrm{d}z = -B_0 Q[1 - \mathrm{e}^{\alpha d_0} - B_1(1-\mathrm{e}^{\alpha d_0}) - B_2\alpha d_0]. \tag{22}$$

In this situation the complex Kerr rotation is given by equation (2), using $k$ from equation (22) with the amplitude reflectance given by the simple reiterative formula for a single, optically homogeneous film.

### *2.3. Case (III). Inhomogeneous semi-infinite film with $Q(z) = Q\,\mathrm{e}^{-\beta z}$*

In this case we deal with a semi-infinite medium whose magnetization, perpendicular to the reflecting surface, decays exponentially with distance from the incident interface with a decay parameter $\beta$. Here $Q$ in equation (20) is replaced by $Q(z) = Q\,\mathrm{e}^{-\beta z}$. The result is remarkably simple and written

$$k = \int \mathrm{d}k = \int_0^\infty \alpha Q B_0\,\mathrm{e}^{-\beta z}\,\mathrm{e}^{\alpha z}\,\mathrm{d}z = -\frac{\mathrm{i}nQ}{(1+n)^2}\frac{\alpha}{(\alpha-\beta)}. \tag{23}$$

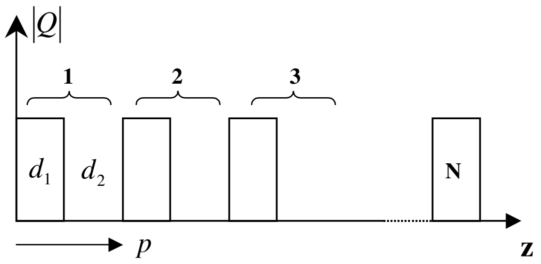


**Figure 2.** Functional dependence of the magnitude of the magneto-optic parameter for an $N$-period multilayer where one layer type is homogeneously magnetized.

### 2.4. Case (IV). Homogeneously magnetized multilayer system

Attention is now drawn to a multilayer system consisting of $N$ periods of two layers (the bi-layer) where the first is homogeneously magnetized with $Q$-parameter $Q_1$. The second is non-magnetic. The situation is illustrated in figure 2. The layer thicknesses are $d_1$ and $d_2$. For the $m$th period the associated Kerr coefficient is

$$\begin{aligned}\Delta k_{m,1} &= \alpha B_0 \int_{z_m}^{z_{m+1}} Q_1 (\mathrm{e}^{\alpha z} + B_1\,\mathrm{e}^{-\alpha z} + B_2)\,\mathrm{d}z \\ &= -\,B_0 Q_1 [(1-\mathrm{e}^{\alpha d_1})\,\mathrm{e}^{\alpha z_m} - B_1(1-\mathrm{e}^{\alpha d_1})\,\mathrm{e}^{-\alpha z_m} + B_2 \alpha d_1].\end{aligned} \tag{24}$$

Consequently, the Kerr coefficient for $N$ periods is obtained by summation over $m$ and is written

$$k_1 = -B_0 Q_1 [(1-\mathrm{e}^{\alpha d_1}) \sum_{m=1}^{m=N} \mathrm{e}^{\alpha z_m} - B_1 (1-\mathrm{e}^{-\alpha d_1}) \sum_{m=1}^{m=N} \mathrm{e}^{-\alpha z_m} - B_2 N \alpha d_1]. \tag{25}$$

Writing

$$f^{\pm} = \sum_{m=1}^{m=N} \mathrm{e}^{\pm \alpha z_m} = (1-\mathrm{e}^{\pm \alpha N p})/(1-\mathrm{e}^{\pm \alpha p}) \tag{26}$$

where the period $p = d_1 + d_2$, it follows that the Kerr coefficient is given by

$$k_1 = -B_0 Q_1 [(1-\mathrm{e}^{\alpha d_1}) f^{+} - B_1 (1-\mathrm{e}^{-\alpha d_1}) f^{-} - B_2 N \alpha d_1]. \tag{27}$$

It should be noted that equation (27) has the same form as equation (22) for $N = 1$ and may be used to obtain the complex Kerr rotation from equation (2) where the reflectance is obtained from the reiterative reflectance formula.

### 2.5. Case (V). Inhomogeneously magnetized multilayer system

As a final case we consider a multilayer system that, while specific in the magnetization and $Q$-parameter profile, is nevertheless of practical importance. The multilayer is constructed from $N$ bi-periods of two materials. The first, of thickness $d_1$, is assumed homogeneous and has a magneto-optic parameter $Q_1$. The other, of thickness $d_2$, is inhomogeneous with a profile that decays from each interface and is described by the function

$$Q_2(z') = Q_2 \frac{\cosh \beta (d_2/2 - z')}{\cosh \beta d_2/2}. \tag{28}$$

It should be noted that the prime added to the $z$-coordinate in equation (28) is to remind the reader that the distance is measured with respect to the beginning of each layer labelled 2.

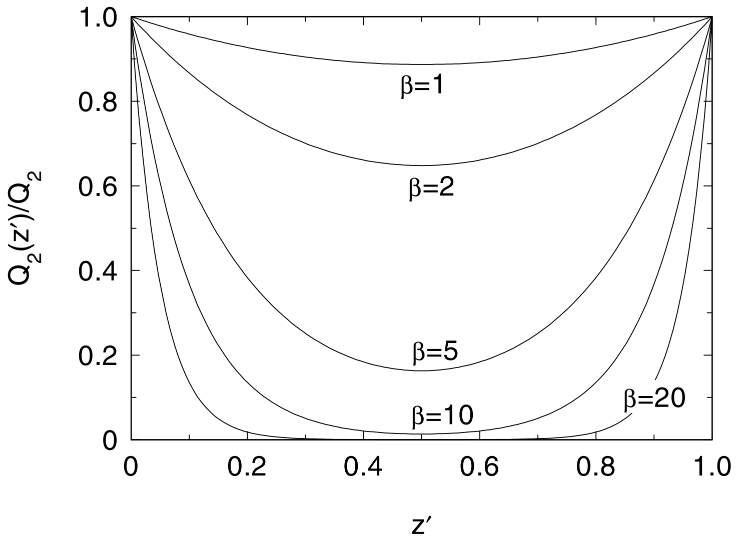


**Figure 3.** Illustration of the variation of the hyperbolic cosine dependence of the magnetization across an inhomogeneous magnetic layer.

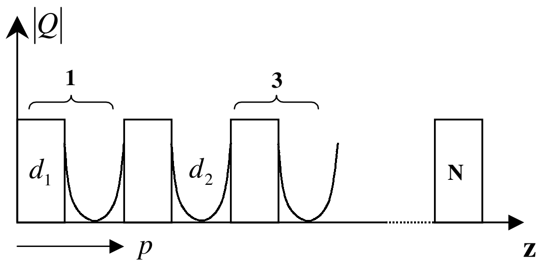


**Figure 4.** Functional dependence of the magnitude of the magneto-optic parameter for an $N$-period multilayer with homogeneous and inhomogeneous magnetic layers.

Recent experiments on CoPd have shown that the moments of Pd atoms are a maximum at the interface and decay exponentially with distance [8]. Where a Pd layer is sandwiched between two Co layers the moment of the Pd has a maximum fixed value at each interface and, in the limit of a thick layer, must decay exponentially from each of the interfaces. The function above satisfies this requirement in that for $\beta d_2 \gg 1$ the function becomes $Q_2(z') = Q_2\,\mathrm{e}^{-\beta z'}$ or $= Q_2\,\mathrm{e}^{+\beta(z'-d_2)}$, depending on which interface is considered. The form of the function is shown in figure 3 for a fixed $d_2 = 1$ but varying values of the decay constant $\beta$.

At first sight, the task of determining the Kerr coefficient for this rather complicated system seems tedious but it must be appreciated that half of the necessary formulation has been derived and is given by equation (27). It only remains to determine the contribution to the Kerr coefficient made by the inhomogeneous layers. In this case, with reference to figure 4, for the $m$th period we have

$$\Delta k_{m,2} = \alpha B_0 \int_{z_m}^{z_{m+1}} Q_2 \frac{\cosh \beta(d_2/2 - z')}{\cosh \beta d_2/2} (\mathrm{e}^{\alpha z} + B_1\,\mathrm{e}^{-\alpha z} + B_2)\,\mathrm{d}z. \tag{29}$$

After integration of equation (29) the result is again summed over $m$ for the $N$ periods. The result is

$$k_2 = -\alpha B_{01}\left\{ Q^+\left[\frac{e^{\alpha d_1} f^+}{\gamma}(1 - e^{\gamma d_2}) - B_1\frac{e^{-\alpha d_1} f^-}{\delta}(1 - e^{-\delta d_2}) - B_2\frac{N}{\beta}(1 - e^{-\beta d_2})\right]\right.$$
$$\left. +Q^-\left[\frac{e^{\alpha d_1} f^+}{\delta}(1 - e^{\delta d_2}) - B_1\frac{e^{-\alpha d_1} f^-}{\gamma}(1 - e^{-\gamma d_2}) + B_2\frac{N}{\beta}(1 - e^{\beta d_2})\right]\right\} \quad (30)$$

where $\gamma = \alpha - \beta$, $\delta = \alpha + \beta$ and

$$Q^{\pm} = \frac{Q_2\, e^{\pm\beta d_2/2}}{2\cosh \beta d_2/2}. \quad (31)$$

Equation (30) represents the contribution of the inhomogeneous layers to the total Kerr component $k$ that, for the completed system is given, using equation (27), by

$$k = k_1 + k_2. \quad (32)$$

Hence, the complex Kerr rotation is

$$\hat{\theta} = \theta + i\varepsilon = \frac{k}{r} = (k_1 + k_2)\left(\frac{r_0 + r_3\, e^{i2\rho}}{1 + r_0 r_3\, e^{i2\rho}}\right)^{-1}. \quad (33)$$

The theoretical analysis outlined above is convenient for dealing with several types of multilayer system and can be used to understand their detailed magneto-optical behaviour. In addition, the formulations may be used to optimize multilayer performance, provided the fundamental material parameters can be measured or are known. To illustrate the value of the method and to demonstrate the reality of inhomogeneous magnetization distributions within magneto-optic media the results of an *in situ* magneto-optic measurement of the dynamic growth during the deposition of a Co–Pd system are summarized and used to obtain the fundamental data that are then used to determine layer thicknesses that optimize magneto-optic performance.

## 3. Experimental example of Co–Pd

*In situ* ellipsometry and Kerr polarimetry, at a wavelength of 633 nm, have been used to follow the continuous evolution of the optical and magneto-optical properties of multiple layers of Co and Pd during their growth. Films were sputter deposited onto a Pd buffer layer on glass substrates up to a maximum of ten bi-layer periods according to the scheme glass/Pd(10)10×(0.3Co/3Pd) (nm). Magnetic hysteresis measurements taken during the deposition process consistently showed strong perpendicular anisotropy at all stages of film growth following the deposition of a single monolayer of Co. It is inappropriate to describe all of the results in full since a full analysis of the extensive data produced during the experiment can be found elsewhere [8]. However, in figure 5 we show the magneto-optic Kerr rotation and ellipticity measured during the deposition of the first three bi-layers. There are several important points that should be noted concerning these curves and the associated optical results.

(1) The ellipsometric data collected throughout the process indicated that the refractive index remained constant during the deposition of the Co and Pd layers and was determined to be $n = (2.33 + i4.11)$.

(2) On the deposition of the Co monolayer there is a large increase in both Kerr rotation and ellipticity. This is several times greater than one would expect on the basis of the magneto-optical properties of Co alone and is a consequence of the polarization of the Pd layer lying below the Co.

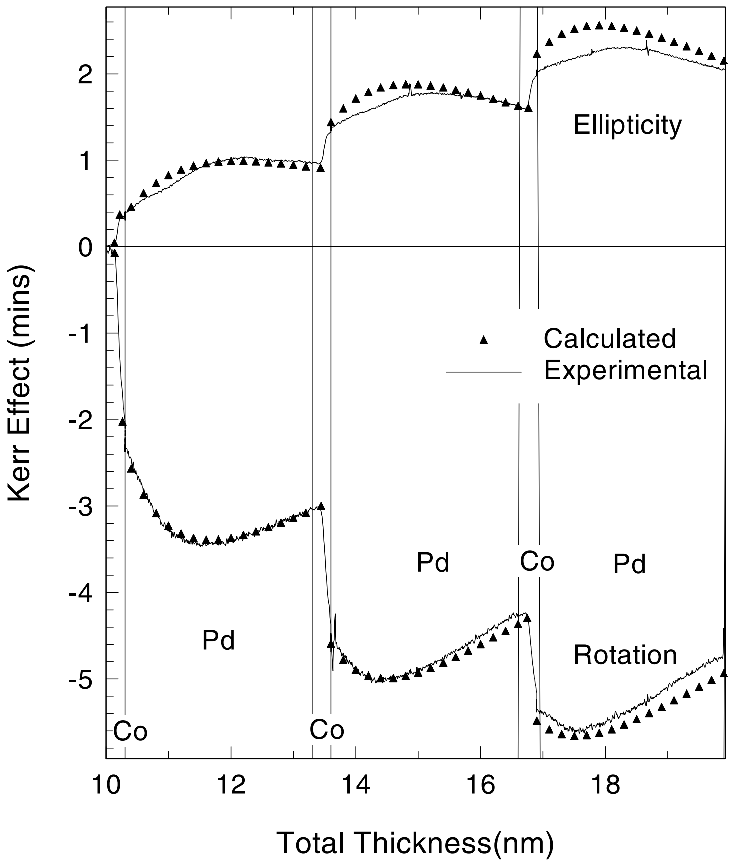


**Figure 5.** Measured and calculated polar Kerr rotation and ellipticity observed during the continuous deposition of three Co–Pd bi-layers on a Pd buffer layer on glass.

(3) On deposition of the Pd, following the Co, there is an initial increase in magneto-optic signal that reaches maximum amplitude before reducing towards zero. This initial increase is a direct consequence of the polarization of the Pd atoms at the upper Co interface. As further Pd atoms are added to the growing layer their magnetic moment is reduced and eventually becomes zero as more atoms are added. Owing to the optical absorption of the Pd the overall Kerr effect is reduced towards zero.

The continuous curves of figure 5 are direct evidence of the inhomogeneous polarization of Pd atoms at interfaces with Co. Furthermore, the markers are points calculated using this analysis using the fundamental $Q$-parameters determined from fitting the curves in the first bi-layer. These parameters were obtained on the assumption that the polarization and, therefore, the $Q$-parameter of the Pd, decreased exponentially with distance from the interfaces with Co. The results for Co and Pd are $Q_1 = -0.0126 + \mathrm{i}0.0007$ and $Q_2 = -0.0025 + \mathrm{i}0.020$, respectively. In addition, the decay constant was found to be $\beta = 0.11$ nm$^{-1}$. The excellence of the agreement between the fitted and experimental curves and the resulting information relating to the fundamental parameters and spatial variation of the magnetic moment of the Pd atoms fully illustrates the usefulness of this analysis. To illustrate further the value of such formulations and the knowledge of various parameters we proceed, using this information, to investigate the

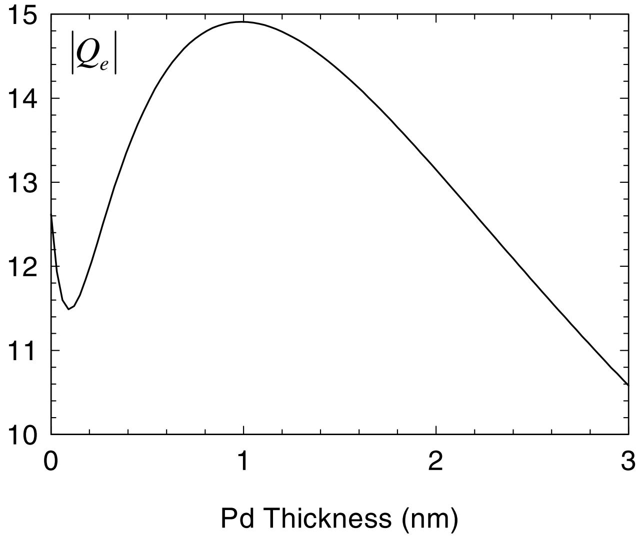


**Figure 6.** Variation of the modulus of the effective magneto-optic parameter of a Co–Pd multilayer as a function of Pd thickness.

optimization of the magneto-optic performance of inhomogeneously magnetized multilayers, specifically using the Co–Pd system as an example.

## 4. Optimization of multilayer performance

In fabricating magnetic multilayer systems for the purposes of magneto-optic recording it is clearly essential to determine the optimum thickness of each elemental layer that will result in the maximum effective magneto-optic parameter of the whole. It should be noted that although the $Q$-parameter may vary on the nanometre scale, the system as a whole may be regarded as having an effective $Q$-parameter and refractive index provided each individual layer satisfies the condition $|nd| \ll \lambda/2\pi$ [19]. In the simple case of a system with homogeneous refractive index it can be shown from [19] that the effective $Q$-parameter is given by the spatial average

$$Q_e = \frac{1}{p}\int_0^p Q(z)\,\mathrm{d}z. \tag{34}$$

If we assume the practical necessity of a single monolayer of Co ($d_1 = 0.3$ nm), in order to polarize the Pd, it remains to determine the thickness of the Pd layer that, when sandwiched between two Co layers, will provide a maximum value of the modulus of the effective $Q$-parameter. It follows logically that the material properties of the whole multilayer system will be the same as each bi-layer. Assuming the spatial variation given by equation (28) the effective value of $Q_e$ for the bi-layer is given by

$$Q_e = \frac{1}{p}\left[Q_1 d_1 + Q_2\int_0^{d_2}\frac{\cosh\beta(d_2/2 - z)}{\cosh\beta d_2 2}\,\mathrm{d}z\right]. \tag{35}$$

Hence,

$$|Q_e| = \frac{1}{p}\left|\left[Q_1 d_1 + Q_2\frac{2}{\beta}\tanh(\beta d_2/2)\right]\right|. \tag{36}$$

Using equation (36) and the physical data obtained for Co and Pd, the variation of the modulus of the effective $Q$-parameter as a function of the thickness of the Pd layer can be calculated. The result is given in figure 6 for a Co layer thickness of 0.3 nm. Clearly the modulus of the effective magneto-optic parameter reaches a maximum corresponding to a Pd layer thickness of 1 nm which closely coincides with that typically found by more tedious empirical methods. This fact fully justifies the analysis that is presented here and verifies the applicability of the fundamental material parameters derived previously from the *in situ* magneto-optical data.

## 5. Summary

An analysis has been presented for dealing with the magneto-optical properties of thin film systems that have inhomogeneously magnetized layers, where the magnetization varies in a direction perpendicular to the film surfaces. It has been assumed that the optical properties can be considered to be homogeneous throughout and whilst this condition is restrictive it is applicable to several multilayered media that are being considered for the purposes of magneto-optic recording. Calculations are based on classical electrodynamic theory using parameters that take into consideration the fundamental thin film optical and magneto-optical constants and their spatial variation in the media. Several simple and relevant cases have been considered in detail which can be compared with existing formulations where the magnetization is homogeneous or where inhomogeneous systems are typical of real systems such as that of the Co–Pd multilayer. The results of an *in situ* experiment monitoring the growth of Co–Pd multilayers by optical and magneto-optical methods have been summarized and compared with calculations and used to derive fundamental material parameters of both homogeneous and inhomogeneous layers. Finally, these material parameters have been used to determine the optimum structures that will lead to maximum performance for the purposes of magneto-optic recording readout.

## References

[1] Mégy R, Bounouh A, Susuki Y, Beauvillain P, Bruno P, Chappert C, Lecuyer B and Veillet P 1995 *Phys. Rev.* B **51** 5586
[2] Katayama T, Suzuki Y, Hayashi M and Geerts W 1994 *J. Appl. Phys.* **75** 6360
[3] Ortega J E, Himpsel F J, Mankey G J and Willis R F 1993 *Phys. Rev.* B **47** 1540
[4] Suzuki Y, Katayama T, Bruno P, Yuasa S and Tamura E 1998 *Phys. Rev. Lett.* **80** 5200
[5] Bruno P, Susuki Y and Chappert C 1996 *Phys. Rev.* B **53** 9214
[6] Uba S, Uba L, Ya Perlov A, Yaresko A N, Antonov V N and Gontarz R 1997 *J. Phys.: Condens. Matter* **9** 447
[7] Zvedzdin A K and Kotov V A 1997 *Modern Magneto-Optics and Magneto-Optic Materials* (Bristol: Institute of Physics) p 447
[8] Atkinson R, Didrichsen G, Hendren W R, Salter I W and Pollard R J *Phys. Rev.* B submitted
[9] Visnovsky S 1991 *Czech. J. Phys.* **41** 663
[10] Atkinson R and Kubrakov N F 1995 *Proc. R. Soc.* A **449** 205
[11] Hashimoto S, Ochiai Y and Aso K 1990 *J. Appl. Phys.* **67** 2136
[12] Sato N 1988 *J. Appl. Phys.* **67** 6424
[13] Zeper W B, Greidanus F J A M, Carcia P F and Fincher C R 1989 *J. Appl. Phys.* **65** 4971
[14] Atkinson R and Lissberger P H 1993 *J. Magn. Magn. Mater.* **118** 271
[15] Gamble R, Lissberger P H and Parker M R 1985 *IEEE Trans. Magn.* **21** 1651
[16] Atkinson R and Lissberger P H 1992 *Appl. Opt.* **31** 6076
[17] Guenther R D and Guenther B D 1990 *Modern Optics* (New York: Wiley)
[18] Lissberger P H 1970 *Rep. Prog. Phys.* **33** 197
[19] Atkinson R 1991 *J. Magn. Magn. Mater.* **95** 61